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ELECTROMAGNETIC FIELD OF THE CIRCULAR MAGNETIC CURRENT LOCATED IN A SEMI-INFINITE BICONICAL SECTION

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The problem of axially-symmetric electromagnetic wave diffraction from the truncated cone placed in the conical region is solved rigorously by mode-matching and analytical regularization techniques. To find the unknown expansion coefficients, the infinite systems of linear algebraic equations (ISLAE) of the second kind are obtained and solved numerically. The analysis of the radiation power dependences on geometrical and frequency parameters are carried out.

Keywords: *analytical regularization, rigorous solution, truncated cone.*

ЕЛЕКТРОМАГНІТНЕ ПОЛЕ ВИТКА МАГНІТНОГО СТРУМУ, РОЗМІЩЕНОГО У НАПІВНЕСКІНЧЕННОМУ БІКОНІЧНОМУ СЕКТОРІ

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Отримано строгий розв’язок осесиметричної задачі дифракції електромагнітного поля на ідеально провідному конусі зі зрізаною вершиною, поміщеному в конічну область. Така структура є моделлю ідеально провідної біконічної поверхні, одне плече якої є напівнескінченний конус, а друге – напівнескінченний конус зі зрізаною вершиною. Задача сформульована у сферичній системі координат і зведена до розв’язання змішаної крайової задачі для рівняння Гельмгольца відносно скалярного потенціалу Дебая. Невідомий скалярний потенціал дифрагованого поля подано у вигляді рядів власних функцій для кожної підобласті, сформованої біконусом. Тобто розв’язок записано у вигляді суми мод, кожна з яких задовольняє граничні умови на конічних поверхнях, умову випромінювання, а також умову обмеженості енергії поля у будь-якому обмеженому об’ємі простору. Остання умова зводиться до виконання умови Мейкснера на краю та вершині конусів. Як джерела випромінювання використано круговий виток синфазного магнітного струму. У такій структурі збуджується поперечна електромагнітна хвиля (ТЕМ-хвиля), поле якої не залежить кутів розхилу конусів, що забезпечує широкосмугові властивості розсіювача. Для розв’язку задачі використано метод спряження тангенціальних компонент поля на сферичній поверхні та співвідношення ортогональності власних функцій. Такий підхід дав змогу звести задачу до нескінченної системи лінійних алгебраїчних рівнянь (НСЛАР) першого роду. Далі застосовано метод аналітичної регуляризації. Показано, що головна частина асимптотики матричних елементів НСЛАР, визначена для великих значень індексів, є оператором типу згортки. Відповідний обернений оператор знайдено в явному вигляді. Отримані оператор типу згортки та його обернений застосовано для зведення задачі до НСЛАР другого роду, яка допускає редукцію за довільних значень параметрів розсіювача. Цю систему використано для знаходження невідомих коефіцієнтів розкладу та відповідних числових розрахунків. Проаналізовано вплив модового складу поля джерела збудження і геометричних параметрів структури на розподіл поля та потужність випромінювання. Виявлено суттєвий вплив вищих ТМ-мод джерела на потужність випромінювання досліджуваної структури.

Ключові слова: *аналітична регуляризація, строгий розв’язок, зрізаний конус.*

Introduction. Biconical structures are widely used in modern radio communications technology. The interest in such structures is caused by the development of ultrashort pulse generation technology and the pressing necessity to develop the means of transmission of the ultrashort pulse using the ultrawideband antennas. The use of bicones as the radiating elements in the radio communication started more than a century ago [1]. The numerous mathematical models that are often used for explaining the phenomena of electromagnetic waves scattering from biconical structures are based on the solution of wave diffraction problem for the perfectly conducting bicone, formed by semi-infinite conical shoulders.

This problem was solved in a spherical coordinate system by separation of variables. The solution was represented as a sum of normal modes, each of which satisfied the boundary condition, as well as the limited energy and radiation conditions. This structure possesses the omnidirection radiation in the azimuth plane of the bicone and provides its wideband properties due to the dominant TEM-wave. The analysis of the electromagnetic wave diffraction from the finite/truncated bicones is based on the field representation by the series of normal modes of subdomains and mode-matching technique application. If the perfectly conducting bicones are under consideration, the orthogonality properties of the meridional functions of the normal waves are used to reduce the problem to the infinite system of linear algebraic equations (ISLAE) to determine the unknown complex amplitudes of the modes.

Convenient theoretical models of a biconical antenna analyzed by the mode-matching techniques were first introduced in [2–5]. The ISLAE, obtained by the mode-matching were solved approximately, without the reduction reasoning. Such method for the biconical structure analysis was widely used in early publications [6]. The main advantage of this method is to consider the form of the scatterer. However, such solutions are formal, because of the singularity of the field components at the edges. Some wave diffraction problems from the finite bicones as well as from the single cones were analyzed by the Wiener–Hopf technique in [7, 8].

In this paper, the analytical regularization technique, early developed in [9–12], was applied to the solution of axially-symmetric electromagnetic wave diffraction from the dissection of the conical region by the truncated semi-infinite conical surface which formed the semi-infinite biconical section. The time dependence is $e^{-i\omega t}$, and it is omitted through the paper.

Formulation of the problem. Let us consider the perfectly conducting bicone $Q = Q_1 \cup Q_2$ in the spherical coordinate system (r,θ,φ) with

$$Q_1 = \{ r \in (0,\infty),\ \theta = \gamma_1;\ \ \varphi \in [0, 2\pi) \},$$
$$Q_2 = \{ r \in (a_1,\infty),\ \theta = \gamma_2;\ \ \varphi \in [0, 2\pi) \},$$

where $\gamma_2 > \gamma_1$, $\gamma_{1(2)} \neq \pi/2$ (see Fig. 1).

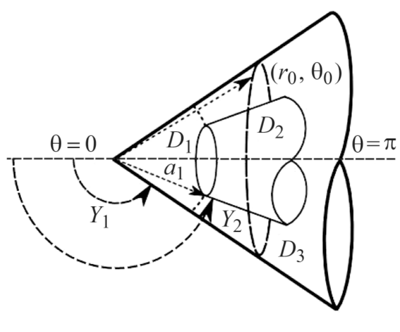

Fig. 1. Geometrical scheme of the bicone.

Let bicone Q be excited by the ring source with magnetic current density as

$$J(r,\theta) = \frac{I_{\varphi}^{(m)} \delta(r - r_0)\delta(\theta - \theta_0)}{r_0 \sin\theta_0}, \tag{1}$$

where $I_\varphi^{(m)}$ is the magnetic current, $\delta(...)$ is the Dirac delta function; r_0, θ_0 are source spherical coordinates, $a_1 < r_0 < \infty$, $\gamma_1 \le \theta_0 \le \gamma_2$. Nonzero field components E_r, E_θ, H_φ excited by source (1), are expressed in terms of H_φ component by

$$E_r = -\frac{1}{\mathrm{i}\omega\varepsilon}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta H_\varphi),$$
$$E_\theta = \frac{1}{\mathrm{i}\omega\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi), \qquad (2)$$

where ε is the dielectric permittivity of the medium.

Taking into account ratio (2), the problem of the electromagnetic field of source (1) diffracted on the bicone Q is reduced to the mixed boundary value problem for Helmholtz equations as

$$\Delta H_\varphi - \frac{H_\varphi}{r^2\sin^2\theta} + k^2 H_\varphi = 0, \qquad (3)$$

where $k = \omega\sqrt{\varepsilon\mu}$ is the wave number, $k = k' + \mathrm{i}k''$; μ is the magnetic permeability; Δ is the Laplace operator,

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right).$$

The unknown H_φ-field satisfies the boundary condition at the bicone Q as

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta(H_\varphi + H_\varphi^i)\right]_{r,\theta\in Q} = 0, \qquad (4)$$

where H_φ^i is the known magnetic component of the incident field produced by source (1).

We search for the solution of the mixed value boundary problem (3), (4) in the class of functions that satisfy the Silver-Muller radiation condition in the form

$$\lim_{r\to\infty} r\left[\vec{i}_r \times \vec{H} + Z^{-1}\vec{E}\right] = 0, \qquad (5)$$

where $Z = \sqrt{\mu/\varepsilon}$ is the medium wave resister, as well as the energy limitation condition as

$$\int_V \left(\varepsilon\,|\,\vec{E}\,|^2 + \mu\,|\,\vec{H}\,|^2\right) dv < \infty. \qquad (6)$$

Here V is any finite volume of integration.

Solution of the problem. For the solution of the diffraction problem let us introduce the regions:

$$D_1 : \{r \in [0, a_1),\ \theta \in (\gamma_1, \pi];\ \varphi \in [0, 2\pi)\},$$
$$D_2 : \{r \in (a_1, \infty),\ \theta \in (\gamma_2, \pi];\ \varphi \in [0, 2\pi)\},$$
$$D_3 : \{r \in (a_1, \infty),\ \theta \in (\gamma_1, \gamma_2);\ \varphi \in [0, 2\pi)\}. \qquad (7)$$

The unknown total magnetic field is presented as

$$H_{\varphi}^{t}(r,\theta)=\begin{cases}\dfrac{\mathrm{i}\omega\varepsilon}{\sqrt{\rho}}\sum\limits_{n=1}^{\infty}x_n^{(1)}\dfrac{\partial}{\partial\theta}P_{z_n-1/2}(-\cos\theta)\dfrac{I_{z_n}(\rho)}{I_{z_n}(\rho_1)}, \\ \qquad (r,\theta)\in D_1 \\ \dfrac{\mathrm{i}\omega\varepsilon}{\sqrt{\rho}}\sum\limits_{n=1}^{\infty}x_n^{(2)}\dfrac{\partial}{\partial\theta}P_{\mu_n-1/2}(-\cos\theta)\dfrac{K_{\mu_n}(\rho)}{K_{\mu_n}(\rho_1)}, \\ \qquad (r,\theta)\in D_2 \\ H_{\varphi}^{i}(\rho,\theta)+\dfrac{\mathrm{i}\omega\varepsilon}{\sqrt{\rho}}\sum\limits_{n=1}^{\infty}x_n^{(3)}\Psi_{\nu_n-1/2}(\cos\theta)\dfrac{K_{\nu_n}(\rho)}{K_{\nu_n}(\rho_1)}. \\ \qquad (r,\theta)\in D_3\end{cases} \tag{8}$$

Here, $x_n^{(1)}, x_n^{(2)}, x_n^{(3)}$ are unknown expansion coefficients; $I_v(\rho)$, $K_v(\rho)$ are modified Bessel and Macdonald functions, respectively; $\rho = sr$, $\rho_1 = sa_1$, $s=-ik$; H_{φ}^{i} is the known field excited by source (1) in the infinite bicone [12];

$$\Psi_{\nu_n-1/2}(\cos\theta)=\begin{cases}\dfrac{1}{\sin\theta}, \ n=1, \\ \dfrac{\partial}{\partial\theta}[R_{\nu_n-1/2}(\cos\theta)], \ n>1,\end{cases}$$

where

$$R_{\nu-1/2}(\cos\theta)=P_{\nu-1/2}(\cos\theta)P_{\nu-1/2}(-\cos\gamma_1)-P_{\nu-1/2}(-\cos\theta)P_{\nu-1/2}(\cos\gamma_1),$$

$P_{\nu-1/2}(\cos\theta)$ is the Legendre function; $\{\nu_n\}_{n=1}^{\infty}$ is the growing sequence of real positive roots of the transcendental equation

$$R_{\nu_n-1/2}(\cos\gamma_2)=0 \tag{9}$$

with $\nu_1 = 1/2$ and $\nu_n \neq n-1/2$ for $n = 2,3,4,\ldots$; $\{z_n\}_{n=1}^{\infty}$, $\{\mu_n\}_{n=1}^{\infty}$ are growing sequences of real positive roots of transcendental equations

$$P_{z_n-1/2}(-\cos\gamma_1)=0,\ P_{\mu_n-1/2}(-\cos\gamma_2)=0. \tag{10}$$

Next, we apply mode-matching for $E_{\theta}^{t}(a_1\pm 0,\theta)$ and $H_{\varphi}^{t}(a_1\pm 0,\theta)$ field components with $\{\ r=a_1, \gamma_1<\theta\leq\pi\ \}$ and use equations (2), (8) to reduce the problem to the solution of ISLAE of the second kind by involving the analytical regularization technique [11]:

$$X - A^{-1}(A-A_{11})X = A^{-1}F. \tag{11}$$

Here, $X=\{x_n\}_{n=1}^{\infty}$ is the unknown vector,

$$x_n = x_n^{(1)}(z_n^2-0.25)P_{z_n-1/2}(-\cos\gamma_2)\ ;$$

A_{11} is the infinite matrix

$$A_{11}:\left\{a_{jn}^{(11)}=\frac{\rho_1 W[K_{\xi_j}, I_{z_n}]_{\rho_1}}{[\xi_j^2-z_n^2]K_{\xi_j}(\rho_1)I_{z_n}(\rho_1)}\right\}_{j,n=1}^{\infty},$$

where $\{\xi_j\}_{j=1}^{\infty} = \{\nu_k\}_{k=1}^{\infty} \bigcup \{\mu_p\}_{p=1}^{\infty}$ is the growing sequence of the roots $\{\nu_k\}_{k=1}^{\infty}$, $\{\mu_p\}_{p=1}^{\infty}$ of the transcendental equations (9) and (10); $F = \{f_j\}_{j=1}^{\infty}$ is the known vector $W[f,\phi]_\rho = f(\rho)\phi'(\rho) - f'(\rho)\phi(\rho)$.

The regularization operators A and A^{-1} are presented as [11, 12]:

$$a_{jm} = (\xi_j - z_m)^{-1}, \tag{12}$$

$$\tau_{nj} = \left\{ \{[M_-(\xi_j)]^{-1}\}'[M_-(z_n)]'(z_n - \xi_j) \right\}^{-1}. \tag{13}$$

Here, $M_-(\nu)$ is the known function. The unique solution of ISLAE (11), which satisfies Meixner condition at the edge and at the vertex of the Q, exists in the class of sequences $x_n = O(n^{-1/2})$ if $n \to \infty$.

Numerical results. All characteristics of the scattered field are calculated by reduction of ISLAE (11). Let us represent the radiation power as

$$W = \lim_{r\to\infty} \frac{1}{2} \int_0^{2\pi} d\varphi \int_{\beta_1}^{\beta_2} E_\theta H_\varphi^* r^2 \sin\theta d\theta .$$

Here, $\beta_1 = \gamma_2$, $\beta_2 = \pi$ and $\beta_1 = \gamma_1$, $\beta_2 = \gamma_2$ correspond to the conical D_2 and the biconical D_3 regions, respectively; H_φ and E_θ by their physical nature, determine the total field in D_2 and the diffracted field in D_3 regions; the upper mark (*) shows the complex conjugate. All numerical analyses are carried out for the fixed coordinates (r_0, θ_0) of the ring source.

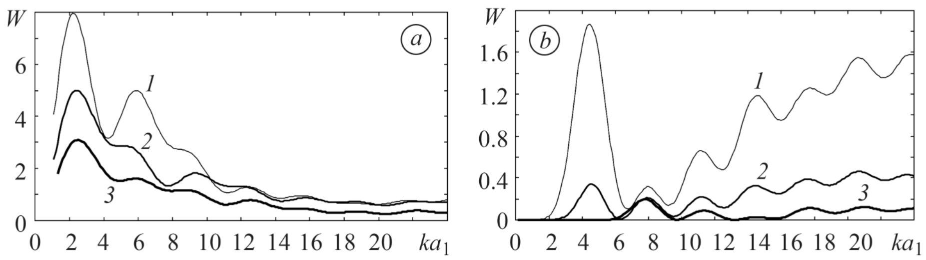

Fig. 2. Radiation power dependences on ka_1 for the excitation of the bicone Q by TEM mode $kr_0 = 25$; $\theta_0 = \gamma_1$, $\gamma_1 = 120°$; *a* – biconical region D_3; *b* – conical region D_2: *1* – $\gamma_2 = 140°$; *2* – $\gamma_2 = 150°$; *3* – $\gamma_2 = 160°$.

Based on the solution of the finite system of linear algebraic equations, we analyze the far-field characteristics for bicone Q with the different geometrical parameters. The curves presented in Fig. 2 show the dependence of the radiated power on the parameter ka_1 if the bicone Q is excided by the TEM mode. We analyze these characteristics for different opening angles γ_2 of the internal truncated cone. From this Figure we observe the effect of resonance scattering, if $2 < ka_1 < 3$ (see Fig. 2*a*) and, if $4 < ka_1 < 5$ (see Fig 2*b*). The behaviour of the curves in Fig. 2 shows the essential influences of the angle γ_2 on the radiation power penetration into the conical and the biconical regions. Next, we analyze the influence of the parameter ka_1 on the radiation power, if our bicone is illuminated by the TEM and TM modes of source (1). This influence is shown in Fig. 3. From the behavior of the curves in this Figure we observe the increase of the radiated power with the growth of the parameter ka_1 for conical and biconical regions.

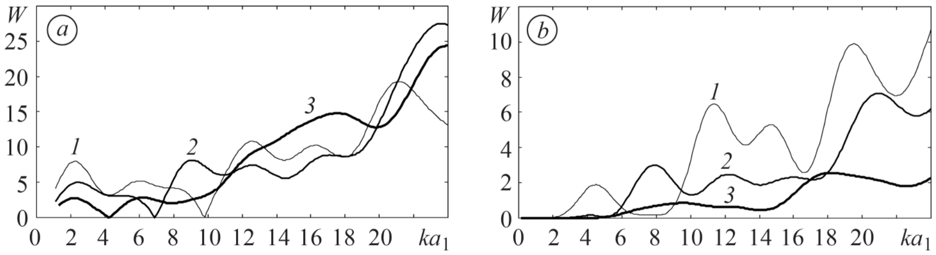

Fig. 3. Radiation power dependences on ka_1 for the excitation of the bicone Q by source (1); $kr_0 = 25$; $\theta_0 = \gamma_1$, $\gamma_1 = 120°$; *a* – biconical region D_3; *b* – conical region D_2: *1* – $\gamma_2 = 140°$; *2* – $\gamma_2 = 150°$; *3* – $\gamma_2 = 160°$.

CONCLUSIONS

The mode-matching and analytical regularization techniques are developed for the solution of canonical diffraction problem of axially-symmetric excitation of the bicone, formed by the truncated and semi-infinite shoulders. The key equation of the second kind (11), the solution of which satisfies all the necessary conditions, is obtained.

The radiation power dependences on geometrical and frequency parameters are analyzed. The essential influence of higher TM modes radiated by the source on the radiation power is observed.

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