UDC 681.78 https://doi.org/10.15407/vidbir2019.47.054

THREE-STEP ELECTRONIC SPECKLE PATTERN INTERFEROMETRY METHOD WITH ARBITRARY PHASE SHIFTS OF REFERENCE WAVE

L. I. Muravsky

Karpenko Physico-Mechanical Institute of the NAS of Ukraine, Lviv

E-mail: muravskyleon@gmail.com

The method of three-step electronic speckle pattern interferometry (ESPI) with arbitrary phase shifts of a reference wave is proposed. This method does not require the calibrated phase shifting techniques, which are necessary for recording speckle interferograms (SI) with fixed phase shifts by conventional temporal ESPI methods. In contrast to the two-step ESPI method with blind phase shift of a reference wave, the proposed three-step method does not use the time consuming procedure of the reference and object wave intensity distribution recording. So, to retrieve the phase map of surface displacements, this method uses only three SI before applying the load to the studied specimen and three SI after applying the load. The proposed method is faster than aforementioned two-step method, because the integrating bucket approach can be used for its realization.

Keywords: *electronic speckle pattern interferometry, arbitrary phase shifts, surface displacement, phase map.*

МЕТОД ТРИКРОКОВОЇ ЕЛЕКТРОННОЇ СПЕКЛ-ІНТЕРФЕРОМЕТРІЇ З ДОВІЛЬНИМИ ФАЗОВИМИ ЗСУВАМИ ОПОРНОЇ ХВИЛІ

Л. І. Муравський

Фізико-механічний інститут ім. Г. В. Карпенка НАН України, Львів

Запропоновано метод трикрокової електронної спекл-інтерферометрії (ЕСІ) з довільними фазовими зсувами опорної хвилі для відтворення поля переміщень поверхні об’єкта досліджень. Він не вимагає використання каліброваних фазових зсувів опорної хвилі, необхідних для реєстрації спекл-інтерферограм (СІ) традиційними методами фазозсувної ЕСІ. На відміну від двокрокового методу ЕСІ у пропонованому відсутня потреба у реєстрації розподілу інтенсивностей предметної та опорної хвиль. Метод також має переваги порівняно з трикроковими методами узагальненої ЕСІ з невідомими однаковими фазовими зсувами. У цих методах віднімають від зареєстрованих СІ усереднений фон, який містить високі просторові частоти, причому частотний спектр фону може суттєво змінюватись після прикладання навантажень до об’єкта, що вносить додаткові похибки у відтворену фазову мапу поля переміщень. Суть методу полягає у прямому визначенні кутів фазових зсувів опорної хвилі в інтерферометрі за допомогою т. зв. кореляційного підходу шляхом обчислення коефіцієнта парної кореляції між двома зареєстрованими СІ, оскільки їх можна розглядати як центровані багатовимірні вектори, компоненти яких є відносними інтенсивностями пікселів у цифрових зображеннях цих інтерферограм. Комп’ютерне моделювання систематичних похибок обчислення фазових зсувів за допомогою коефіцієнта парної кореляції показало, що ці похибки зменшуються зі збільшенням шорсткості поверхні. А оскільки усі методи ЕСІ придатні лише для оптично шорстких поверхонь, то кореляційний підхід ефективніший для ЕСІ, ніж методи фазозсувної інтерферометрії для відтворення оптично гладких поверхонь матеріалів. У методі за трьома СІ з попередньо обчисленими довільними фазовими зсувами опорного променя визначають різницю фаз у кожному пікселі двох фазових мап поверхні до і після прикладання навантаження, розрахованих за зареєстрованими СІ. В результаті отримають шукану фазову мапу двомірного поля переміщень поверхні після прикладання навантаження. Метод дає можливість плавно змінювати фазу опорного променя для забезпечення високої швидкості реєстрації СІ.

Ключові слова: *електронна спекл-інтерферометрія, довільні фазові зсуви, переміщення поверхні, фазова мапа.*

Introduction. Electronic speckle pattern interferometry (ESPI) techniques became efficient tools to study the deflected mode of structural materials and members [1, 2]. The conventional phase-shifting ESPI techniques are based on given exact phase shifts of a reference wave. Obviously, phase shifts in multiples of $\pi/2$ or equal to $2\pi/N$, where N is an integer [1, 2], are the most widespread among them. In order to realize these techniques, calibrated phase shifters should be used to obtain the given or the same arbitrary phase shifts and to generate not less than three speckle interferograms (SIs) for each fixed studied surface state, which leads to labor-consuming procedure of displacement field retrieval.

One of possible solution of this problem consists in producing series of speckle interferograms differing by arbitrary phase shifts of a reference wave. For example, some phase-step methods used two fringe patterns with $\pi/2$ shift for initial and deformed surface conditions [3, 4], though such shifts should be produced by a calibrated phase shifters. On the other hand, Sesselman and Gonsalves [5] proposed the technique, in which any given phase shift except those close to a multiple of π, can be used. But this technique also demands implementation of a calibrated phase shifter to provide the predetermined phase step. Another method of two-step phase-shifting ESPI with blind phase shift of the reference wave was proposed to provide the full-field retrieval of a surface displacement map by producing only two SIs of initial and deformed surfaces and two similar SIs differing by blind phase shift of a reference wave [6]. It can be implemented for phase shifts ranging from 0 to π in contrast to the two-step phase-shifting technique with small unknown phase shift (i.e. $\leq \pi/3$) [7]. This method realizes recording of two SIs before deformation of the studied surface, namely, the initial SI $s_1(k, l)$ and SI $s_2(k, l)$ with the reference wave shifted on an arbitrary angle α, and two SIs after the surface deformation, namely, SI $s_3(k, l)$ without phase shift of a reference wave and SI $s_4(k, l)$ with phase shift of the reference wave on the same angle α, where k, l is the image pixel coordinate. The spectral and correlation approaches to the blind phase shift extraction were used in this method. Note that the correlation approach applied to the two-step ESPI technique was adopted from the two-step phase-shifting interferometry technique with blind shift of a reference wave developed by Muravsky et al. [8, 9]. However, to eliminate the background and retrieve the searched displacement phase map by these techniques, it is necessary to record additionally the spatial distributions of the reference and object wave.

In this paper, we propose the new method of three-step ESPI with arbitrary phase shifts, which excludes recording of a reference and object waves to eliminate the background. The correlation approach is used to calculate unknown blind phase shifts in this method.

Method of three-step ESPI with arbitrary phase shifts. Let's consider three SI of the structural material optically rough surface area. These SI are recorded before applying the load or at the intermediate stage of loading. SIs are recorded in two-beam interferometer with a normal incidence of the object wave on the studied target. Twyman–Green or Michelson interferometers can be used for this purpose. Each SI differs from other SIs only in an arbitrary phase shift relative to the object wave, and the initial phase of the reference wave in the first SI is zero. Therefore, the spatial distribution of these SIs recorded by computer can be expressed as

$$\left.\begin{aligned} i_1(k,l) &= i'(k,l)\left[1+V(k,l)\cos\psi(k,l)\right] \\ i_2(k,l) &= i'(k,l)\left\{1+V(k,l)\cos\left[\psi(k,l)+\alpha_{21}\right]\right\} \\ i_3(k,l) &= i'(k,l)\left\{1+V(k,l)\cos\left[\psi(k,l)+\alpha_{31}\right]\right\} \end{aligned}\right\}, \qquad (1)$$

where $i'(k, l) = i_o(k, l) + i_r(k, l)$ is the background intensity distribution represented as a sum of the object and reference beams intensities, respectively, $V(k,l) = \dfrac{2\sqrt{i_o(k,l)i_r(k,l)}}{i_o(k,l)+i_r(k,l)}$ is the fringe visibility, $\psi(k, l)$ is the spatial phase distribution of the rough surface before deformation, k, l are the pixel numbers in digital speckle interferograms recorded by a computer, and α_{21}, α_{31} are the arbitrary (blind) phase shifts of the reference wave relative to its initial position with a phase equal to zero.

If the arbitrary phase shifts α_{21} and α_{31} are unknown, the set of equations (1) can't be solved. In order to find these phase shifts, we can use the correlation approach proposed and developed by Muravsky et al. [8, 9]. According to this approach, phase shifts are calculated using pair correlation between two SIs, that is

$$\alpha_{21} = \arccos\frac{\left\langle\left[i_1(k,l)-\left\langle i_1(k,l)\right\rangle\right]\left[i_2(k,l)-\left\langle i_2(k,l)\right\rangle\right]\right\rangle}{\sigma_{i_1(k,l)}\sigma_{i_2(k,l)}}, \tag{2}$$

$$\alpha_{31} = \arccos\frac{\left\langle\left[i_1(k,l)-\left\langle i_1(k,l)\right\rangle\right]\left[i_3(k,l)-\left\langle i_3(k,l)\right\rangle\right]\right\rangle}{\sigma_{i_1(k,l)}\sigma_{i_3(k,l)}}, \tag{3}$$

where $\sigma_{i_1(k,l)}$, $\sigma_{i_2(k,l)}$, $\sigma_{i_3(k,l)}$ are the RMS of the intensity distributions in SIs $i_1(k, l)$, $i_2(k, l)$, $i_3(k, l)$, respectively.

After deformation of the studied specimen, the intensity spatial distributions of three SIs can be written as

$$\left.\begin{aligned} i_4(k,l) &= i''(k,l)\left\{1+V'(k,l)\cos\left[\psi(k,l)+\Delta\varphi(k,l)\right]\right\} \\ i_5(k,l) &= i''(k,l)\left\{1+V'(k,l)\cos\left[\psi(k,l)+\Delta\varphi(k,l)+\alpha_{54}\right]\right\} \\ i_6(k,l) &= i''(k,l)\left\{1+V'(k,l)\cos\left[\psi(k,l)+\Delta\varphi(k,l)+\alpha_{64}\right]\right\} \end{aligned}\right\}, \tag{4}$$

where $i''(k, l) = i'_o(k, l) + i_r(k, l)$ is the background intensity distribution and $i'_o(k, l)$ is the object wave intensity distribution after applying the load, $V'(k,l) = \dfrac{2\sqrt{i'_o(k,l)i_r(k,l)}}{i'_o(k,l)+i_r(k,l)}$ is the fringe visibility after applying the load, $\Delta\varphi(k, l)$ is the searched spatial distribution of the displacement phase map in each pixel of the studied surface area, and α_{54}, α_{64} are the arbitrary (blind) phase shifts of the reference wave relative to its initial position with a phase equal to zero. As in the conventional ESPI [2], we suppose that spatial distribution of the surface phase $\psi(k, l)$ before and after applying the load remains unchanged due to the absence of the surface microrelief changes.

The arbitrary phase shift angles α_{54} and α_{64} are determined in the same way as angles α_{21} and α_{31}, that is

$$\alpha_{54} = \arccos\frac{\left\langle\left[i_4(k,l)-\left\langle i_4(k,l)\right\rangle\right]\left[i_5(k,l)-\left\langle i_5(k,l)\right\rangle\right]\right\rangle}{\sigma_{i_4(k,l)}\sigma_{i_5(k,l)}}, \tag{5}$$

$$\alpha_{64} = \arccos\frac{\left\langle\left[i_4(k,l)-\left\langle i_4(k,l)\right\rangle\right]\left[i_6(k,l)-\left\langle i_6(k,l)\right\rangle\right]\right\rangle}{\sigma_{i_4(k,l)}\sigma_{i_6(k,l)}}, \tag{6}$$

where $\sigma_{i_4(k,l)}$, $\sigma_{i_5(k,l)}$, $\sigma_{i_6(k,l)}$ are the RMS of the intensity distributions in SIs $i_4(k, l)$, $i_5(k, l)$, $i_6(k, l)$, respectively.

In order to find the searched displacement phase field $\Delta\varphi(k, l)$, it is necessary to extract the phase surface $\psi(k, l)$ before deformation from the set of equations (1) and to extract the phase surface $[\psi(k, l) + \Delta\varphi(k, l)]$ after deformation from the set of equations (4). With this purpose, we transform the sets of three equations (1) and (4) into sets of two equations omitting the pixel coordinates (k, l) for brevity, that is

$$\left.\begin{aligned} \frac{i_2 - i'}{i_1 - i'} &= \frac{\cos\left(\psi + \alpha_{21}\right)}{\cos\psi} = \cos\alpha_{21} - \tan\psi\sin\alpha_{21} \\ \frac{i_3 - i'}{i_1 - i'} &= \frac{\cos\left(\psi + \alpha_{31}\right)}{\cos\psi} = \cos\alpha_{31} - \tan\psi\sin\alpha_{31} \end{aligned}\right\}, \tag{7}$$

$$\left.\begin{aligned} \frac{i_5 - i''}{i_4 - i''} &= \frac{\cos\left(\psi + \Delta\varphi + \alpha_{54}\right)}{\cos\left(\psi + \Delta\varphi\right)} = \cos\alpha_{54} - \tan\left(\psi + \Delta\varphi\right)\sin\alpha_{54} \\ \frac{i_6 - i''}{i_4 - i''} &= \frac{\cos\left(\psi + \Delta\varphi + \alpha_{64}\right)}{\cos\left(\psi + \Delta\varphi\right)} = \cos\alpha_{64} - \tan\left(\psi + \Delta\varphi\right)\sin\alpha_{64}, \end{aligned}\right\}. \tag{8}$$

Solving the systems of equations (7) and (8) with respect to phase ψ, we obtain:

$$\tan\psi = -\frac{c_1}{b_1}, \tag{9}$$

where

$$b_1 = \left(i_1 - i_3\right)\sin\alpha_{21} + \left(i_2 - i_1\right)\sin\alpha_{31},$$

$$c_1 = \left(i_2 - i_3\right) + \left(i_3 - i_1\right)\cos\alpha_{21} + \left(i_1 - i_2\right)\cos\alpha_{31},$$

and

$$\tan\left(\psi + \Delta\varphi\right) = -\frac{c_2}{b_2}, \tag{10}$$

where

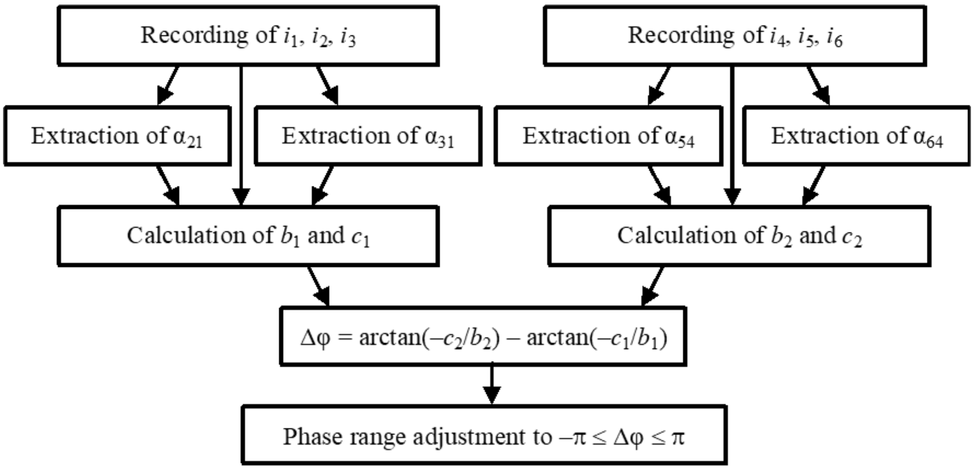
$$b_2 = \left(i_4 - i_6\right)\sin\alpha_{54} + \left(i_5 - i_4\right)\sin\alpha_{64},$$

$$c_2 = \left(i_5 - i_6\right) + \left(i_6 - i_4\right)\cos\alpha_{54} + \left(i_4 - i_5\right)\cos\alpha_{64}.$$

Thus, the wrapped phase map $\Delta\varphi(k, l)$ of the displacement phase field in each $(k, l)^{\text{th}}$ pixel of the optically rough surface is given as

$$\begin{aligned} &\Delta\varphi(k,l) = \left[\psi(k,l) + \varphi(k,l)\right] - \psi(k,l) = \\ &= \arctan\left[-\frac{c_2(k,l)}{b_2(k,l)}\right] - \arctan\left[-\frac{c_1(k,l)}{b_1(k,l)}\right]. \end{aligned} \tag{11}$$

The obtained wrapped PM can be considered as the first stage of the displacement field retrieval. A flow graph of the first stage of the method implementation is shown in the figure. This stage is completed by the retrieval of a wrapped PM $\Delta\varphi(k, l)$ with a phase range adjustment to $-\pi \leq [\Delta\varphi_w(k, l)] \leq \pi$. To carry out this adjustment, we jump from the final values (c_1/b_1) and (c_2/b_2), which are the input parameters in equation (11), to two pairs of input parameters c_1, b_1 and c_2, b_2, that are the functions of the sine and cosine values, respectively. This jump can be implemented by using, for example the “atan2” function [10]. Retrieval of a final PM of the studied object from the wrapped PM $\Delta\varphi_w(k, l)$ can be finished by the standard unwrapping procedure.



Flow graph of the proposed method.

CONCLUSION

The new method of three-step ESPI with arbitrary phase shifts of a reference wave is proposed. In contrast to the similar two-step ESPI method [6], this method does not require registration of spatial distributions of the reference and object waves to eliminate the background. In this method, the systematic errors for calculation of blind phase shifts α_{21}, α_{31} and α_{54}, α_{64} do not exceed similar errors for the aforementioned two-step ESPI method. The method allows using the integrating bucket approach [11], due to which the speed of SIs recording before and after applying the load raises. Therefore, this method is more suitable for the fast reconstruction of in-plane and out-of-plane surface displacements in comparison with the two-step ESPI method.

1. Yang, L.; Ettemeyer, A. Strain Measurement by Three-Dimensional Electronic Speckle Pattern Interferometry: Potentials, Limitations, and Applications. *Opt. Eng.* **2003,** *42,* 1257-1266.
2. Sirohi, R. S. *Optical Methods of Measurement: Wholefield Techniques*, 2nd ed.; Taylor & Francis Group: Boca Raton, FL, London, New York, 2009.
3. Almazan-Cuellar, S.; Malacara-Hernandez, D. Two-Step Phase-Shifting Algorithm. *Opt. Eng.* **2003,** *42,* 3524-3531.
4. Yu, Q.; Fu, S.; Liu, X.; Yang, X.; Sun, X. Single-Phase-Step Method with Contoured Correlation Fringe Patterns for ESPI. *Opt. Express*. **2004,** *12,* 4980-4985.
5. Sesselmann, M.; Gonçalves Jr., A. A. Single Phase-Step Algorithm for Phase Difference Measurement Using ESPI. *Proc. SPIE—Int. Soc. Opt. Eng.* **1998,** *3478*, 153-158.
6. Muravsky, L. I.; Kmet’, A. B.; Voronyak, T. I. Two Approaches to the Blind Phase Shift Extraction for Two-Step Electronic Speckle Pattern Interferometry. *Opt. Eng*. **2013,** *52,* 101909.
7. Huang, Y. H.; Hung, S. Y.; Janabi-Sharifi, F.; Wang, W.; Liu, Y. S. Quantitative Phase Retrieval in Dynamic Laser Speckle Interferometry. *Opt. Lasers Eng*. **2012,** *50*, 534-539.
8. Muravsky, L. I.; Ostash, O. P.; Kmet’, A. B.; Voronyak, T. I.; Andreiko, I. M. Two-Frame Phase-Shifting Interferometry for Retrieval of Smooth Surface and Its Displacements. *Opt. Lasers Eng*. **2011,** *49*, 305-312.
9. Muravsky, L. I.; Kmet’, A. B.; Voronyak, T. I. Retrieving the Relief of Low-Roughness Surface Using a Two-Step Interferometric Method with Blind Phase Shift of a Reference Wave. *Opt. Lasers Eng*. **2012,** *50*, 1508-1516.
10. Wikipedia, the Free Encyclopedia. atan2. https://en.wikipedia.org/wiki/Atan2 (accessed Nov 20, 2019).
11. Greivenkamp, J. E. Generalized Data Reduction for Heterodyne Interferometry. *Opt. Eng*. **1984,** *23*, 350-352.

Received 17.09.2019