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MODEL OF MULTICOMPONENT NARROW-BAND PERIODICALLY NON-STATIONARY RANDOM SIGNAL

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The correlation and spectral properties of a multicomponent narrow-band periodical non-stationary random signal (PNRS) and its Hilbert transformation are considered. It is shown that multicomponent narrowband PNRS differs from the monocomponent signal. This difference is caused by correlation of the quadratures for the different carrier harmonics. Such features of the analytic signal must be taken into account when using the Hilbert transform for the analysis of real time series.

Keywords: *polycomponent narrow-band periodically non-stationary random signal, Hilbert transform, analytic signal.*

МОДЕЛЬ БАГАТОКОМПОНЕНТНОГО ВУЗЬКОСМУГОВОГО ПЕРІОДИЧНО НЕСТАЦІОНАРНОГО ВИПАДКОВОГО СИГНАЛУ

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Розглянуто періодичний нестаціонарний випадковий сигнал (ПНВС), що є суперпозицією стохастично модульованих за амплітудою та фазою гармонік з кратними частотами. Спектри модуляцій вважали зосередженими в смугах, ширина яких є меншою за базову частоту сигналу. Математичну модель коливань у вигляді такого ПНВС широко використовують у різних областях науки і техніки: в статистичній радіофізиці, в теорії зв’язку і телеметрії, в технічній діагностиці, геофізиці та океанології, в статистичній гідроакустиці тощо. Визначення параметрів модуляцій у вібраціях пошкоджених механізмів дає можливість виявити та знайти характерні риси дефектів на ранніх стадіях їх розвитку. Вузькосмугові модуляції можна успішно проаналізувати, використавши перетворення Гільберта. Розглянуто кореляційні та спектральні властивості багатокомпонентного вузькосмугового ПНВС та його перетворення Гільберта. Отримано формули для коефіцієнтів Фур’є авто- та взаємокореляційних функцій, спектральних густин сигналу і їх перетворень Гільберта, знайдені зв’язки між ними. Проаналізовано кореляційну та спектральну структури аналітичного сигналу. Виявлено, що на відміну від однокомпонентного вузькосмугового сигналу аналітичний сигнал є періодично нестаціонарним випадковим процесом, при цьому кількість гармонік його кореляційної функції вдвічі менша, ніж кореляційної функції сигналу. Встановлено, що амплітуди гармонік кореляційної функції однокомпонентного вузькосмугового сигналу визначаються тільки кореляціями між квадратурами різних номерів гармонік. Отримані результати є теоретичною базою для використання перетворення Гільберта під час розв’язування практичних задач.

Ключові слова: *багатокомпонентний вузькосмуговий періодично нестаціонарний випадковий сигнал, перетворення Гільберта, аналітичний сигнал.*

The methods of the covariance and spectral analysis of periodically non-stationary random processes (PNRP) are widely used in the different areas of science and technology, including radiophysics, geophysics, acoustics, vibrodiagnostics, etc. [1–5]. The possibility of joint consideration of different types of the stochastic recurrence on the PNRP basis is provided by their harmonic representation [1, 2, 4]:

$$\xi(t)=\sum_{k\in Z}\xi_k(t)e^{ik\omega_0 t}, \tag{1}$$

where $\xi_k(t)$ are jointly stationary random processes; $\omega_0=\frac{2\pi}{P}$; P is the non-stationary period, and $\mathbf{Z}$ is the set of integer numbers. Each amplitude- and phase-modulated harmonic of series (1) can be considered as monocomponent signal $\xi(t)$ and it can be analyzed using the Hilbert transform. The harmonic modulation can be low- and high-frequency in comparison with harmonic frequency. The high-frequency modulation occurred, for example, when we considered vibration of rolling bearing with a local defect [3]. So called envelope analysis is one of the most successful techniques for diagnostics of faults in rolling bearing elements [6, 7]. The low-frequency modulation also occured in vibrations during crack initiation [4, 8]; as well as in series of stochastic oscillations of other physical nature [4, 9]. The aim of this paper is to investigate the properties of the Hilbert transform of a low-frequency narrow-band periodically non-stationary random signal (PNRS) and related to it analytic signal set in the form (1).

Assume that series (2) is limited, the number of harmonics is equal to N. Putting $\xi_0(t)\equiv 0$, we have:

$$\xi(t)=\sum_{\substack{k=-N\\k\neq 0}}^{N}\xi_k(t)e^{ik\omega_0 t}=\sum_{k=1}^{N}\left[\xi_k^c(t)\cos k\omega_0 t+\xi_k^s(t)\sin k\omega_0 t\right]. \tag{2}$$

Assume also that the values of the power spectral densities of the modulating processes $\xi_k(t)$ are concentrated in the interval $\left[-\frac{\omega_0}{2},\frac{\omega_0}{2}\right]$. Such PNRS we call a narrow-band PNRS. The conditions of the Bedrosian theorem are satisfied for narrow-band PNRS and for its Hilbert transform we have:

$$\eta(t)=\sum_{k=1}^{N}\left[\xi_k^c(t)\sin k\omega_0 t-\xi_k^s(t)\cos k\omega_0 t\right]=i\sum_{k=1}^{N}\left[\overline{\xi}_k(t)e^{-ik\omega_0 t}-\xi_k(t)e^{ik\omega_0 t}\right]. \tag{3}$$

To simplify the further analysis rewrite (2) in the similer form:

$$\xi(t)=\sum_{k=1}^{N}\left[\xi_k(t)e^{ik\omega_0 t}+\overline{\xi}_k(t)e^{-ik\omega_0 t}\right]. \tag{4}$$

Then for the covariance function obtain:

$$b_\xi(t,u)=\mathrm{E}\,\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)=2\,\mathrm{Re}\left\{S_1(u,t)+S_2(u,t)\right\}, \tag{5}$$

where

$$S_1(u,t)=\sum_{k,l=1}^{N}R_{kl}^{(\xi)}(u)e^{i(l-k)t}e^{il\omega_0 u}, \tag{6}$$

$$S_2(u,t)=\sum_{k,l=1}^{N}R_{\xi_k\xi_l}(t,u)e^{i(l+k)t}e^{il\omega_0 u}, \tag{7}$$

$$R_{kl}^{(\xi)}(u)=\mathrm{E}\,\overline{\overset{\circ}{\xi}_k}(t)\overset{\circ}{\xi}_l(t+u)=\frac{1}{4}\left[R_{kl}^{c}(u)+R_{kl}^{c}(u)-i\left[R_{kl}^{cs}(u)-R_{kl}^{sc}(u)\right]\right], \tag{8}$$

$$R_{\xi_k \xi_l}(u) = \mathrm{E}\,\overset{\circ}{\xi}_k(t)\overset{\circ}{\xi}_l(t+u) = \frac{1}{4}\Big[R_{kl}^{c}(u) - R_{kl}^{c}(u) - i\big[R_{kl}^{cs}(u) + R_{kl}^{sc}(u)\big]\Big], \qquad (9)$$

and $R_{kl}^{c}(u) = \mathrm{E}\,\overset{\circ}{\xi}{}_k^{c}(t)\overset{\circ}{\xi}{}_l^{c}(t+u)$; $R_{kl}^{s}(u) = \mathrm{E}\,\overset{\circ}{\xi}{}_k^{s}(t)\overset{\circ}{\xi}{}_l^{s}(t+u)$; $R_{kl}^{cs}(u) = \mathrm{E}\,\overset{\circ}{\xi}{}_k^{c}(t)\overset{\circ}{\xi}{}_l^{s}(t+u)$; $\overset{\circ}{\xi}{}_k^{c}(t) = \xi_k^{c}(t) - m_k^{c}$; $\overset{\circ}{\xi}{}_k^{s}(t) = \xi_k^{s}(t) - m_k^{s}$; $m_k^{c} = \mathrm{E}\xi_k^{c}(t)$; $m_k^{s} = \mathrm{E}\xi_k^{s}(t)$. Introduce in sum (6) new indices of summations $z = l - k$:

$$S_1(u,t) = \sum_{l=1}^{N} e^{il\omega_0 u} \sum_{r=l-1}^{l-N} R_{l-r,l}^{(\xi)}(u) e^{ir\omega_0 t}.$$

Now change the order of summation (Fig. 1)

$$S_1(u,t) = \sum_{r=-N+1}^{0} e^{ir\omega_0 t} \sum_{l=1}^{r+N} R_{l-r,l}^{(\xi)}(u) e^{il\omega_0 u} + \sum_{r=1}^{N-1} e^{ir\omega_0 t} \sum_{l=r+1}^{N} R_{l-r,l}^{(\xi)}(u) e^{il\omega_0 u}. \qquad (10)$$

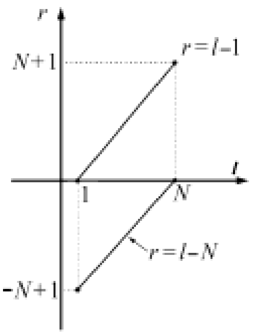

Fig. 1. Changing the order of summation.

The second sum (7) after introducing a new index $r = l + k$ is represented in the following form:

$$S_2(u,t) = \sum_{l=1}^{N} e^{il\omega_0 u} \sum_{r=l+1}^{l+N} R_{\xi_{r-l}\xi_l}(u) e^{ir\omega_0 t}.$$

Changing the order of summation we yield:

$$S_2(u,t) = \sum_{r=2}^{N+1} e^{ir\omega_0 t} \sum_{l=1}^{r-1} R_{\xi_{r-l}\xi_l}(u) e^{il\omega_0 u} + \sum_{r=N+2}^{2N} e^{ir\omega_0 t} \sum_{l=r-N}^{N} R_{\xi_{r-l}\xi_l}(u) e^{il\omega_0 u}. \qquad (11)$$

Proceeding from (10) and (11), we can find the expressions for the coefficients of the covariance function (5) in the Fourier sense representation:

$$b_{\xi}(t,u) = \sum_{r=-2N}^{2N} B_r^{(\xi)}(u) e^{ir\omega_0 t} = B_0^{(\xi)}(u) + \sum_{r=1}^{2N}\Big[C_r^{(\xi)}(u)\cos 2\omega_0 t + S_r^{(\xi)}(u)\sin 2\omega_0 t\Big].$$

Here $B_r^{(\xi)}(u) = \frac{1}{2}\Big[C_r^{(\xi)}(u) - iS_r^{(\xi)}(u)\Big]$. Taking into account (8) and (9) for the $zero^{th}$ covariance component we have

$$B_0^{(\xi)}(u) = \frac{1}{2}\sum_{l=1}^{N}\Big[\Big[R_{ll}^{c}(u) + R_{ll}^{s}(u)\Big]\cos l\omega_0 u + \tilde{\tilde{R}}_{ll}^{cs}(u)\sin l\omega_0 u\Big],$$

where $\tilde{\tilde{R}}_{ll}^{cs}(u)$ is the odd part of the cross-covariance function $R_{ll}^{cs}(u)$. Hence

$$f_0^{(\xi)}(\omega) == \frac{1}{4}\sum_{l=1}^{N}\Big[f_{ll}^{c}(\omega + l\omega_0) + f_{ll}^{s}(\omega + l\omega_0) + f_{ll}^{c}(\omega - l\omega_0) + f_{ll}^{s}(\omega - l\omega_0) + + 2\Big[\tilde{\tilde{f}}_{ll}^{cs}(\omega + l\omega_0) - \tilde{\tilde{f}}_{ll}^{sc}(\omega - l\omega_0)\Big]\Big] \tag{12}$$

herewith

$$f_{kl}^{c,s}(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{kl}^{c,s}(u)\, e^{-i\omega u} d\omega, \quad f_{kl}^{cs}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{kl}^{cs}(u)\, e^{-i\omega u} d\omega,$$

and $\tilde{\tilde{f}}_{ll}^{cs}(\omega)$ is the odd part of $f_{ll}^{cs}(\omega)$. It may be seen from expression (12), that the bandwidth of the PNRS zero component in the considered case is limited by interval $\left[-N\omega_0 - \frac{\omega_0}{2},\, N\omega_0 + \frac{\omega_0}{2}\right]$. Since

$$\left| f_{kl}^{cs}(\omega)\right|^2 \le f_{kl}^{c}(\omega)\, f_{kl}^{s}(\omega), \tag{13}$$

it follows from (12) and (13) that the bandwidth of any spectral component $f_r(\omega)$ cannot be larger than the bandwidth of $f_0(\omega)$.

For the covariance component of the non-zeroth number we obtain:

$$C_r^{(\xi)}(-u) = \frac{1}{2}\sum_{l\in M_1}^{N}\Big[\Big[R_{r-l,l}^{c}(u) - R_{r-l,l}^{s}(u)\Big]\cos l\omega_0 u + \Big[R_{r-l,l}^{cs}(u) - R_{r-l,l}^{sc}(u)\Big]\sin l\omega_0 u\Big],$$

$$S_r^{(\xi)}(u) = \frac{1}{2}\sum_{l\in M_1}^{N}\Big[\Big[R_{r-l,l}^{cs}(u) - R_{r-l,l}^{sc}(u)\Big]\cos l\omega_0 u - \Big[R_{r-l,l}^{c}(u) - R_{r-l,l}^{s}(u)\Big]\sin l\omega_0 u\Big],$$

where $M_1 = \{1,...,r-1\}$ for $r = \overline{2, N+1}$ and $M_2 = \{r-N,...,N\}$ for $r = \overline{N+2, 2N}$. Then

$$B_r^{(\xi)}(u) = \frac{1}{4}\sum_{l\in M_1}^{N}\Big[R_{r-l,\,l}^{c}(u) - R_{r-l,\,l}^{s}(u) - i\Big[R_{r-l,\,l}^{cs}(u) - R_{r-l,\,l}^{sc}(u)\Big]e^{il\omega_0 u}\Big], \tag{14}$$

and

$$f_r^{(\xi)}(\omega) = \frac{1}{2}\sum_{l\in M_1}^{N}\left[f_{r-l,\,l}^{c}(\omega - l\omega_0) - f_{r-l,\,l}^{s}(\omega - l\omega_0) - i\left[\begin{matrix} f_{r-l,\,l}^{cs}(\omega - l\omega_0) + \\ + f_{r-l,\,l}^{sc}(\omega - l\omega_0)\end{matrix}\right]\right], \tag{15}$$

It can be seen from (15) that the values of the spectral components $f_r^{(\xi)}(\omega)$ as $r \ge N$ belong to interval $[0, r\omega_0]$.

<u>Proposition 1.</u> The covariance components of the narrow-band PNRS and its Hilbert transform whose numbers $r \ge N$ differ only by sign $B_r^{(\eta)}(u) = -B_r^{(\xi)}(u)$ and their cross-covariance components are symmetric odd lag functions and they are defined by the equality $B_r^{(\xi\eta)}(u) = B_r^{(\eta\xi)}(u) = -iB_r^{(\xi)}(u)$.

<u>Proof.</u> Taking into account the equalities for the spectral $f_r^{(\xi,\eta)}(\omega)$ and cross-spectral $f_r^{(\xi\eta)}(\omega)$ component of signal (4) and its Hilbert transform (3) [10, 11]:

$$f_r^{(\eta)}(\omega) = \mathrm{H}(\omega)\, f_r^{(\eta\xi)}(\omega)\,,$$

$$f_r^{(\eta\xi)}(\omega) = -\mathrm{H}(\omega)\, f_r^{(\eta)}(\omega),$$

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and the properties of the spectral components [4] we can rewrite the expressions for the covariance components

$$B_r^{(\eta)}(u) = \int_0^{\infty} f_r^{(\eta)}(\omega) e^{i\omega u} d\omega,$$

$$B_r^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} f_k^{(\xi\eta)}(\omega) e^{i\omega u} d\omega\,, \quad B_r^{(\eta\xi)}(u) = \int_{-\infty}^{\infty} f_r^{(\eta\xi)}(\omega) e^{i\omega u} d\omega$$

in the following form

$$B_r^{(\eta)}(u) = \int_{-\infty}^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega - 2\int_0^{r\omega_0} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega,$$

$$B_r^{(\eta\xi)}(u) = -i\left[\int_0^{\infty} f_r^{(\xi)}(\omega + r\omega_0) e^{-i\omega u} d\omega + \int_0^{r\omega_0} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega - \int_{r\omega_0}^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega\right],$$

$$B_r^{(\xi\eta)}(u) = i\int_0^{\infty}\left[f_r^{(\xi)}(\omega + r\omega_0) e^{-i\omega u} - f_r^{(\xi)}(\omega) e^{i\omega u}\right] d\omega.$$

For the spectral component with number $r \geq N$

$$f_r^{(\xi)}(\omega) = \begin{cases} f_r^{(\xi)}(\omega), & \omega \in [0, r\omega_0], \\ 0, & \omega \notin [0, r\omega_0]. \end{cases}$$

Then

$$\int_0^{\infty} f_r^{(\xi)}(\omega + r\omega_0) e^{-i\omega u} d\omega = 0,$$

$$\int_{r\omega_0}^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega = 0,$$

and hence

$$B_r^{(\eta)}(u) = -\int_0^{r\omega_0} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega = -\int_{-\infty}^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega.$$

$$B_r^{(\xi\eta)}(u) = B_r^{(\eta\xi)}(u) = -i\int_0^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega = -i\int_{-\infty}^{\infty} f_r^{(\xi)}(\omega) e^{i\omega u} d\omega.$$

Thus, equality $B_r^{(\eta)}(u) = -B_r^{(\xi)}(u)$ holds true. ■

Similar results can be derived by the direct calculation of the auto-covariance $B_r^{(\eta)}(u)$ and cross-covariances $B_r^{(\xi\eta)}(u)$, $B_r^{(\eta\xi)}(u)$ on the basis of relations (3) and (4). For the covariance functions we have:

$$b_\eta(t,u) = \mathrm{E}\eta(t)\eta(t+u) = 2\,\mathrm{Re}\{S_1(u,t) - S_2(u,t)\}, \tag{16}$$

$$b_{\xi\eta}(t,u) = \mathrm{E}\overline{\xi}(t)\eta(t+u) = 2\,\mathrm{Im}\{S_1(u,t) + S_2(u,t)\}, \tag{17}$$

$$b_{\eta\xi}(t,u) = \mathrm{E}\overline{\eta}(t)\xi(t+u) = 2\,\mathrm{Im}\{S_1(u,t) - S_2(u,t)\}. \tag{18}$$

Equalities $B_0^{(\eta)}(u) = B_0^{(\xi)}(u)$ and $B_2^{(\eta)}(u) = -B_2^{(\xi)}(u)$ follow from (16) for $r \geq N$ immediately. The $zero^{th}$ cross-covariance component is the summand of the sum $S_1(u,t)$. Therefore $B_0^{(\eta\xi)}(u) = -B_0^{(\xi\eta)}(u)$ and

$$B_0^{(\xi\eta)}(u) = \frac{1}{2}\sum_{l=1}^{N}\Big[\Big[R_{l\,l}^{c}(u) + R_{l\,l}^{s}(u)\Big]\sin l\omega_0 u - 2\tilde{\tilde{R}}_{l,l}^{cs}(u)\cos l\omega_0 u\Big], \tag{19}$$

where $\tilde{\tilde{R}}_{l,l}^{cs}(u)$ is the odd part of the covariance function $R_{l,l}^{cs}(u)$. It is easily seen from equation (19) that $B_0^{(\xi\eta)}(u)$ is the odd lag function. Proceeding from (19) we obtain the expression for the $zero^{th}$ cross-spectral component

$$f_0^{(\xi\eta)}(\omega) = \frac{i}{4}\sum_{l=1}^{N}\Big[\Big[f_{ll}^{c}(\omega - l\omega_0) + f_{ll}^{s}(\omega - l\omega_0) - f_{ll}^{c}(\omega + l\omega_0) - f_{ll}^{s}(\omega + l\omega_0)\Big] +$$

$$+2\Big[f_{ll}^{cs}(\omega + l\omega_0) + f_{ll}^{sc}(\omega - l\omega_0)\Big]\Big]$$

which allows us to conclude that in this case the equation

$$B_0^{(\xi\eta)}(u) = -\int_0^{\infty} f_0^{(\xi)}(\omega)\sin\omega u d\omega$$

is also true.

The cross-covariance components $B_r^{(\xi\eta)}(u), r \geq N,$ are the summand of the sum $S_2(u,t),$ therefore equality $B_r^{(\xi\eta)}(u) = B_r^{(\eta\xi)}(u)$ follows from (17) and (18) immediately. The expression for $B_r^{(\xi\eta)}(u)$ we obtain from (11):

$$B_r^{(\xi\eta)}(u) = -\frac{1}{4}\sum_{l\in M}^{N}\Big[R_{r-l,\,l}^{cs}(u) + R_{r-l,\,l}^{sc}(u) + i\Big[R_{r-l,\,l}^{c}(u) - R_{r-l,\,l}^{s}(u)\Big]\Big]e^{il\omega_0 u}. \tag{20}$$

The comparison of (14) and (20) reveals that $B_r^{(\xi\eta)}(u) = -iB_r^{(\xi)}(u)$ for $r \geq N$.

Now we can formulate the following proposition.

<u>Proposition 2.</u> If the covariance functions of the quadratures of the individual harmonics for PNRS satisfy the conditions

$$R_{kl}^{c}(u) + R_{kl}^{s}(u) \neq 0 \text{ or } R_{kl}^{cs}(u) - R_{kl}^{sc}(u) \neq 0$$

for numbers $k \neq l,$ the analytic signal $\zeta(t) = \xi(t) + i\eta(t)$ is periodically non-stationary and its covariance function is defined by the series

$$b_\zeta(t,u) = \sum_{r=-N+1}^{N-1} B_r^{(\zeta)}(u)e^{ir\omega_0 t},$$

where

$$B_0^{(\zeta)}(u) = \sum_{l=1}^{N}\Big[\Big[R_{ll}^{c}(u) + R_{ll}^{s}(u)\Big]\cos l\omega_0 u + \tilde{\tilde{R}}_{ll}^{cs}(u)\sin l\omega_0 u +$$

$$+ i\Big[R_{ll}^{c}(u) + R_{ll}^{s}(u)\Big]\sin l\omega_0 u - \tilde{\tilde{R}}_{ll}^{cs}(u)\cos l\omega_0 u\Big]$$

and

$$B_r^{(\xi\eta)}(u) = \sum_{l\in M_2}^{N}\Big[R_{l-r,\,l}^{c}(u) + R_{l-r,\,l}^{s}(u) - i\Big[R_{l-r,\,l}^{cs}(u) - R_{l-r,\,l}^{sc}(u)\Big]\Big]e^{il\omega_0 u}, \tag{21}$$

herewith the set $M_2 = \begin{cases} \{1,....,r+N\} & for\ r \le N, \\ \{r+1,...,N\} & for\ r < N, \end{cases}$ and the values of the spectral components lie outside the interval $[0, r\omega_0]$.

Proof. Taking into account equalities $B_2^{(\eta)}(u) = -B_2^{(\xi)}(u)$ and $B_r^{(\xi\eta)}(u) = B_r^{(\eta\xi)}(u)$ for $r \ge N$ we come to the conclusion that series for the covariance function contains only $N-1$ harmonics. We obtain the expressions for the covariance components using the relations (5)–(7) and (16)–(18) and

$$b_\zeta(t,u) = b_\xi(t,u) + b_\eta(t,u) + i\left[b_{\xi\eta}(t,u) - b_{\eta\xi}(t,u)\right].$$

We have

$$b_\xi(t,u) + b_\eta(t,u) = 2\sum_{r=-N+1}^{N-1}\sum_{l\in M_1}\Big[\left[R_{l-r,l}^{c}(u) + R_{l-r,l}^{s}(u)\right]\cos\left(r\omega_0 t + l\omega_0 u\right) +$$
$$+\left[R_{l\,l}^{cs}(u) + R_{l\,l}^{sc}(u)\right]\sin\left(r\omega_0 t + l\omega_0 u\right)\Big],$$

$$b_{\xi\eta}(t,u) - b_{\eta\xi}(t,u) = 2\sum_{r=-N+1}^{N-1}\sum_{l\in M_2}\Big[\left[R_{l-r,l}^{c}(u) + R_{l-r,l}^{s}(u)\right]\sin\left(r\omega_0 t + l\omega_0 u\right) -$$
$$-\left[R_{l\,l}^{cs}(u) - R_{l\,l}^{sc}(u)\right]\cos\left(r\omega_0 t + l\omega_0 u\right)\Big].$$

Hence

$$b_\zeta(t,u) = \sum_{r=-N+1}^{N-1} e^{ir\omega_0 t}\sum_{l\in M_2}\left[R_{l-r,\,l}^{c}(u) + R_{l-r,\,l}^{s}(u) - i\left[R_{l-r,\,l}^{cs}(u) - R_{l-r,\,l}^{sc}(u)\right]\right]e^{il\omega_0 u}.$$

Note that we obtain the same formula for the covariance function using the series for the analytic signal

$$\zeta(t) = \xi(t) + i\eta(t) = \sum_{k=1}^{N}\begin{bmatrix}\xi_k^c(t)\left(\cos k\omega_0 t + i\sin k\omega_0 t\right) - \\ -i\left[\xi_k^s(t)\left(\cos k\omega_0 t + i\sin k\omega_0 t\right)\right]\end{bmatrix} = 2\sum_{k=1}^{N}\xi(t)e^{ik\omega_0 t}.$$

Proceeding from (21) for the spectral components of the analytic signal we get

$$f_r^{(\zeta)}(\omega) = \sum_{l\in M_2}\left[f_{l-r,\,l}^{c}(\omega - l\omega_0) + f_{l-r,\,l}^{s}(\omega - l\omega_0) - i\begin{bmatrix} f_{l-r,\,l}^{cs}(\omega - l\omega_0) - \\ -f_{l-r,\,l}^{sc}(\omega - l\omega_0)\end{bmatrix}\right]. \tag{22}$$

The values of the summation index $l = \overline{1, N+r}$ for $r \le 0$ and $l = \overline{r+1, N}$ $l = \overline{1, N+r}$ for $r > 0$, i.e. for all l, inequality $l \ge r$ is satisfied. In this case, as it follows from (22), the values of the spectral components $f_r^{(\zeta)}(\omega)$ lie outside the interval $[0, r\omega_0]$ for $\forall r = \overline{-N+1, N-1}$. ■

The variance of the analytic signal is equal to

$$b_\zeta(t,0) = \sum_{r=-N+1}^{N-1} B_r^{(\zeta)}(0)e^{ir\omega_0 t} =$$
$$= B_0^{(\zeta)}(0) + 2\sum_{r=1}^{N-1}\sum_{l=r+1}^{N}\Big[\left[R_{l-r,\,l}^{c}(0) + R_{l-r,\,l}^{s}(0)\right]\cos r\omega_0 t + \left[R_{l-r,\,l}^{cs}(0) - R_{l-r,\,l}^{sc}(0)\right]\sin r\omega_0 t\Big],$$

herewith $B_r^{(\zeta)}(0)=\sum_{l=1}^{N}\left[R_l^c(0)+R_l^s(0)\right]$. The degree of the periodical nonstationarity of the analytic signal

$$\gamma=\frac{\sum_{r=1}^{N-1}\left|B_r^{(\zeta)}(0)\right|}{B_0^{(\zeta)}(0)}$$

is not equal to zero only in the cases when the cross-covariance functions of the quadrature components for different numbers are not equal to zero at point $u=0$.

The obtained results show that the properties of analytic signal for polycomponent PNRP differ from that for monocomponent signal. It is periodically non-stationary random process. This non-stationarity is caused only by correlation of the quadrature for the different carrier harmonics. Such features of the analytic signal must be taken into account when using the Hilbert transform for the analysis of real time series.

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