МАТЕМАТИЧНІ МОДЕЛІ СИГНАЛІВ ТА СИСТЕМ

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THE COVARIANCE ANALYSIS OF THE PERIODICALLY NON-STATIONARY RANDOM SIGNAL WITH NARROW-BAND MODULATION OF CARRIER HARMONICS

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The periodically non-stationary random signals (PNRSs), whose carrier harmonics are modulated by jointly stationary high-frequency random processes are analyzed. A repre-sentation of the signal in the form of a superposition of high-frequency components is obtained and it is shown that these components are jointly periodically non-stationary random processes. The random process is periodically non-stationary of the second order only in the case when some of the cross-covariance functions of its modulation processes are not equal to zero. The correlations of the PNRP spectral harmonics and the correlations of the modulating processes in series representation are equivalent. Evaluating the specific features of the auto- and cross-covariances for modulating processes as well as contribution of each pair to the covariance component values allows us to detect defects at early stages.

Keywords: *periodically non-stationary random processes, vibration signal, covariance analysis.*

КОРЕЛЯЦІЙНИЙ АНАЛІЗ ПЕРІОДИЧНО НЕСТАЦІОНАРНОГО ВИПАДКОВОГО СИГНАЛУ З ВУЗЬКОСМУГОВОЮ МОДУЛЯЦІЄЮ НЕСУЧИХ ГАРМОНІК

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Проаналізовано періодично нестаціонарні випадкові сигнали (ПНВС), несучі гармоніки яких змодульовані взаємостаціонарними високочастотними випадковими процесами. Сигнали подано у вигляді суперпозиції високочастотних складових і встановлено, що ці складові є взаємоперіодичними нестаціонарними випадковими процесами. Випадковий процес є періодично нестаціонарним другого порядку лише тоді, коли деякі із взаємокореляційних функцій його модуляційних процесів не дорівнюють нулю. Оцінка специфічних характеристик авто- та взаємокореляційних функцій для модулюючих процесів і внеску кожної пари індивідуальних кореляцій у кореляційні компоненти сигналу загалом дає можливість виявити дефекти на ранніх стадіях розвитку. Коливання, що характеризуються одночасно повторюваністю та стохастичністю, описують моделями у вигляді періодично нестаціонарних випадкових процесів (ПНВП) на основі стохастичної модуляції несучих гармонік. Ця модуляція може бути низько- і високочастотною, а також широко- або вузькосмуговою у кожному конкретному випадку. Оскільки високочастотну модуляцію зумовлюють локальні несправності в елементах обертових машин, то основну увагу зосереджено саме на аналізі цього випадку. Вібраційний сигнал можна описати як суперпозицію стохастично амплітудно- і фазово- модульованих несучих гармонік з декількома частотами. Таке подання є характерне для ПНВП. При цьому симптоматичною ознакою появи пошкоджень механізму є власне періодична нестаціонарність вібраційного сигналу. Тому необхідно детально проаналізувати кореляційну та спектральну структури стохастичних модуляцій несучих гармонік ПНВП. Проаналізовано вузькосмуговий модульований, багатокомпо-нентний ПНВС, поданий у вигляді ряду з високочастотними модуляційними складовими, і доведено, що вони є стаціонарними та взаємоперіодично нестаціонарними випадковими процесами. Встановлено, що сума їх автокореляцій визначає кореляційну функцію стаціонарної апроксимації сигналу, а сума їх взаємокореляцій рівна змінним у часі складовим кореляції сигналу, котрі власне і визначають нестаціонарність вібраційного сигналу.

Ключові слова: *періодично нестаціонарні випадкові процеси, вібраційний сигнал, кореляційний аналіз.*

Introduction. Oscillations, which are characterized by their recurrence and stochasticity, as well as interactions between these features are described by oscillation models in the form of periodically non-stationary random processes (PNRPs) based on stochastic modulation of their carrier harmonics [1–3]. This modulation may be low- or high-frequency, and may be wide- or narrow-band, in each domain. The authors in [4] found that the vibrational properties of both low- and high-frequency modulations can occur in different cases. Since high-frequency modulation is caused by the appearance of local faults in the elements of rotating machines, the main attention in the literature was focused on an analysis of this effect. This analysis was traditionally carried out using so-called "envelope" or "high-resonance" techniques (also known as "demodulated resonance analysis") [5–8] before the cyclostationary approach was developed [7–16].

The envelope analysis was devised as an empirical technique [5, 7, 17]. It should be applied to a purely random part of the signal, and hence the deterministic components must be canceled. Several procedures were developed for this purpose, including time synchronous (coherent) averaging [1–3, 18], linear prediction [19], self-adaptive noise cancellation [20], the spectral method [21] and others. Although the best results are obtained using time-synchronous averages, this approach requires a separate operation, including individual resampling, for each considered case. To avoid these disadvantages, the component or least squares methods can be used to separate the deterministic parts of the signal. These approaches do not require interpolation or resampling, and can be applied to arbitrary sampling steps which satisfy the anti-aliasing conditions.

Vibration signals could be represented as products of the low-frequency, modulating, deterministic signals and random forced oscillations. However, a more detailed analysis of the covariance structure for the vibration of damaged rotating machinery showed that the results of repetitive mechanical impacts are more complicated [10, 16]. The vibration signal in many cases can be described as a superposition of stochastically amplitude- and phase-modulated carrier harmonics with multiple frequencies. Note that this representation is a characteristic feature of PNRP [1, 22]. Taking this into consideration, we can analyze in more detail the covariance and spectral structures of stochastical modulations of PNRP carrier harmonics, since the periodical non-stationarity is the symptomatic feature for the appearance of damage [4].

In this paper we will analyze the narrow-band modulated, multi-component PNRS, give its representation in the form of a series of high-frequency components and prove that they are stationary and jointly periodically non-stationary random processes; we will show that the sum of their auto-covariances determines the covariance function of stationary approximation of the signal, and summing up of their cross-covariances yields the signal covariance terms which change with time.

Model of PRRP as stochastically modulated signal. The PNRP mean function $m_\xi(t)=E\xi(t)$, where E is the operator of the mathematical expectation, the covariance functions $b_\xi(t,u)=E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)$, $\overset{\circ}{\xi}(t)=\xi(t)-m_\xi(t)$, are periodical functions of time, i.e. $m_\xi(t)=m_\xi(t+P)$, $b_\xi(t,u)=b_\xi(t+P,u)$, where P is period. If $m_\xi(t)$ are absolutely integrable time functions over interval $[0,P]$, namely

$$\int_0^P |m_\xi(t)| dt < \infty, \quad \int_0^P |b_\xi(t,u)| dt < \infty \ \forall u \in R,$$

then they can be represented in the form of a Fourier series as follows:

$$m_\xi(t) = \sum_{k \in Z} m_k e^{ik\omega_0 t} = m_0 + \sum_{k \in N} \left(m_k^c \cos k\omega_0 t + m_k^s \sin k\omega_0 t \right), \quad (1)$$

$$b_\xi(t,u) = \sum_{k \in Z} B_k^{(\xi)}(u) e^{ik\omega_0 t} = B_0^{(\xi)}(u) + \sum_{k \in N} \left[C_k^{(\xi)}(u) \cos k\omega_0 t + S_k^{(\xi)}(u) \sin k\omega_0 t \right], \quad (2)$$

where $\omega_0 = \frac{2\pi}{P}$, $m_k \frac{1}{2}\left(m_k^c - i m_k^s\right)$, $B_k^{(\xi)}(u) = \frac{1}{2}\left[C_k^{(\xi)}(u) - i S_k^{(\xi)}(u)\right] \forall k \neq 0$, Z is the set of integer numbers, N is the set of natural numbers and

$$m_k = \frac{1}{P} \int_0^P m_\xi(t) e^{-ik\omega_0 t} dt,$$

$$B_k^{(\xi)} = \frac{1}{P} \int_0^P b_\xi(t,u) e^{-ik\omega_0 t} dt.$$

The mean function in Eq. (1) describes the deterministic part of the vibrations, which is usually associated with the macroscopic defects of mechanical systems, such as imbalance, eccentricity, misalignment, etc. The stochastic part $\overset{\circ}{\xi}(t)$ contains information about the non-linearity and non-stationarity of the vibration signal caused by friction forces, changes in the viscosity of lubricants, surface irregularities, etc. An analysis of the stochastic part, including its periodical non-stationarity characteristics, i.e. the Fourier coefficients $B_k^{(\xi)}(u)$ in Eq. (2), allows defects to be detected in the early stages after their initiation [10, 12, 16].

The covariance components $B_k(\tau)$ satisfy the equality:

$$B_k^{(\xi)}(-u) = B_k^{(\xi)}(u) e^{-ik\omega_0 u}.$$

The zeroth covariance component is an even function: $B_0^{(\xi)}(-u) = B_0^{(\xi)}(u)$. It is also a positive definite function [1, 23, 24]. Thus $B_0^{(\xi)}(u)$ has all the properties of the covariance function of stationary random processes. Therefore, this quantity is called a covariance function of stationary approximation of PNRP [1, 16, 23].

If

$$\int_{-\infty}^{\infty} |b_\xi(t,u)| du < \infty,$$

then we can introduce the function

$$f_\xi(\omega,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} b_\xi(t,u) e^{-i\omega_0 u} du,$$

which is called the instantaneous spectral density of PNRP. Taking into account the Fourier series in Eq. (2), we have

$$f_\xi(\omega,t) = \sum_{k \in Z} f_k^{(\xi)}(\omega) e^{ik\omega_0 t},$$

where

$$f_k^{(\xi)}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} B_k^{(\xi)}(u)e^{-i\omega u}du\,. \tag{3}$$

The spectral density Fourier coefficients are called the cyclic spectrum [3, 9] or spectral components [16, 23]. They satisfy the equalities:

$$f_k^{(\xi)}(-\omega)=f_k^{(\xi)}(\omega+k\omega_0)\ ,$$

$$f_{-k}^{(\xi)}(\omega)=\overline{f}_k^{(\xi)}(\omega+k\omega_0)\ .$$

Here "¯" signifies complex conjugation. The zeroth spectral component $f_0^{(\xi)}(\omega)$ is a real even non-negative function:

$$f_0^{(\xi)}(\omega)=\overline{f}_0^{(\xi)}(\omega)\,,\quad f_0^{(\xi)}(-\omega)=f_0^{(\xi)}(\omega)\,,\quad f_0^{(\xi)}(\omega)\geq 0\,.$$

As the Fourier transform of the covariance function of PNRP stationary approximation, it determines the PNRP spectral composition.

The non-zero[th] spectral components $f_k^{(\xi)}(\omega)$ determine the correlations of harmonics in the PNRP spectral representation

$$\xi(t)=\int_{-\infty}^{\infty} e^{i\omega t}dz(\omega)\,, \tag{4}$$

where

$$Edz(\omega)=\sum_{k\in Z} m_k\delta(\omega-k\omega_0)d\omega\,,$$

$$E\overline{d\overset{\circ}{z}(\omega_1)}\,d\overset{\circ}{z}(\omega_2)=\sum_{k\in Z} f_k(\omega_1)\delta(\omega_2-\omega_1+k\omega_0)d\omega_1\,d\omega_2\ , \tag{5}$$

and $d\overset{\circ}{z}(\omega)=dz(\omega)-m_k\delta(\omega-k\omega_0)d\omega$, $\delta(\omega)$ is the Dirac delta function. It follows from Eq. (5) that the periodical non-stationarity of random processes stipulates in Eq. (4) the correlations harmonics shifted by $k\omega_0$.

Proceeding from the PNRP series representation [1, 22, 25],

$$\xi(t)=\sum_{k\in Z}\xi_k(t)\,e^{ik\omega_0 t}\,, \tag{6}$$

where $\xi_k(t)$ are jointly stationary random processes, we can deduce that the properties of the mean in Eq. (1) and covariance function in Eq. (2) are determined by the properties of modulating processes $\xi_k(t)$. The mathematical expectations of $\xi_k(t)$ are equal to the Fourier coefficients of the mean function $m(t): E\xi_k(t)=m_k$. The cross-covariance functions $R_{kl}(u)=E\overline{\overset{\circ}{\xi}_k(t)}\overset{\circ}{\xi}_l(t+u)$, where $\overset{\circ}{\xi}_k(t)=\xi_k(t)-m_k$, determine the Fourier coefficients of the PNRP covariance function with the number $k=l-r$:

$$B_k^{(\xi)}(u)=\sum_{l\in Z} R_{l-k,l}^{(\xi)}(u)e^{il\omega_0 u}\,. \tag{7}$$

It follows from Eq. (7) that the random process in Eq. (6) is the second order periodically non-stationary only in the case when some of the cross-covariance functions of the modulation processes are not equal to zero. The zero[th] covariance component is defined by the auto-covariance functions of $\xi_l(t)$:

$$B_0^{(\xi)}(u)=\sum_{l\in Z} R_{ll}^{(\xi)}(u)e^{-il\omega_0 u}.$$

Substituting Eq. (7) into Eq. (3), we obtain the equality:

$$f_k^{(\xi)}(\omega)=\sum_{l\in Z} f_{l-k,l}^{(\xi)}(\omega-l\omega_0), \qquad (8)$$

where

$$f_{kl}^{(\xi)}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R_{kl}^{(\xi)}(u)e^{-i\omega u}du.$$

It follows from Eqs. (5) and (8) that the correlations of the PNRP spectral harmonics and the correlations of the modulating processes in series representation in Eq. (6) are equivalent.

Covariation and spectral properties of PNRP. Thus, within the framework of the PNRP theory of the second order, the properties of vibration are described by the mean function, covariance function, instantaneous spectral density and their Fourier coefficient. The properties of the vibration deterministic part can be specified by the amplitude $A_k=\left[(m_k^c)^2+(m_k^s)^2\right]^{1/2}$ and phase $\varphi_k=\operatorname{arctg}\frac{m_k^s}{m_k^c}$ spectra. The spectral composition of the vibration's stochastic part is defined by the zero[th] spectral component $f_0^{(\xi)}(\omega)$. Integrating this quantity we get the time-averaged value of the power of the vibration stochastic part:

$$B_0^{(\xi)}(0)=\int_{-\infty}^{\infty} f_0^{(\xi)}(\omega)d\omega. \qquad (9)$$

The power time changes of the stochastic part are determined by the variance Fourier coefficients:

$$B_k^{(\xi)}(0)=\int_{-\infty}^{\infty} f_k^{(\xi)}(\omega)d\omega. \qquad (10)$$

The quantities in Eq. (10) are the summary characteristics for the correlations of harmonics shifted by $k\omega_0$. They are complex, so we can describe the power time changes $b_\xi(t,0)$ using amplitude $V_k=\left[\left[C_k^{(\xi)}(0)\right]^2+\left[S_k^{(\xi)}(0)\right]^2\right]^{1/2}$ and phase $\varphi_k=\operatorname{arctg}\frac{S_k^{(\xi)}(0)}{C_k^{(\xi)}(0)}$ spectra. Note that it is not expedient to calculate these spectra using the transforms in Eqs. (9) and (10), since they can be calculated directly on the basis of experimental data using cyclic (component) statistics [25].

The covariance component with number k can also be considered as the summary characteristics of the cross-covariances of the modulating processes, the indexes of which differ by number k. Assuming that the series in Eq. (6) is finite and the number of harmonics is equal to L, we have:

$$B_k^{(\xi)}(u) = \sum_{l \in S} R_{l-k,l}^{(\xi)}(u) e^{il\omega_0 u}, \qquad (11)$$

where $S = \{-L, ..., k+L\}$ for $k \leq 0$ and $S = \{k-L, ..., L\}$ for $k > 0$. The cross-covariance functions $R_{kl}^{(\xi)}(u)$ are determined by the relation:

$$R_{kl}^{(\xi)}(u) = \frac{1}{4}\left[R_{\xi_k \xi_l}^c(u) + R_{\xi_k \xi_l}^s(u) - i\left[R_{\xi_k \xi_l}^{cs}(u) - R_{\xi_k \xi_l}^{sc}(u)\right]\right], \qquad (12)$$

where

$$R_{\xi_k \xi_l}^{c,s}(u) = E\,\overset{\circ}{\xi}{}_k^{c,s}(t)\,\overset{\circ}{\xi}{}_l^{c,s}(t+u), \quad R_{\xi_k \xi_l}^{cs}(u) = E\,\overset{\circ}{\xi}{}_k^{c}(t)\,\overset{\circ}{\xi}{}_l^{s}(t+u),$$

$$\overset{\circ}{\xi}{}_k^{c,s}(t) = \xi_k^{c,s}(t) - m_k^{c,s}, \quad m_k^{c,s} = E\xi_k^{c,s}(t).$$

It follows from Eqs. (11) and (12) that the individual modulated harmonics in the representation in Eq. (1)

$$\varepsilon_k(t) = \xi_k^c(t)\cos k\omega_0 t + \xi_k^s(t)\sin k\omega_0 t$$

are periodically non-stationary and jointly periodically non-stationary random processes.

For $u = 0$, we have

$$B_k(0) = \sum_{l \in S} R_{l-k,l}^{(\xi)}(0).$$

So, the Fourier coefficients of the signal variance are determined by the elements of the quadratic matrix

$$R(0) = \left[R_{kl}^{(\xi)}(0)\right]_{2L \times 2L}.$$

Since $R_{kl}^{(\xi)}(u) = \bar{R}_{lk}^{(\xi)}(-u)$, this matrix is Hermitian. Summing up the elements of the diagonal for $k = l$, we obtain the zero$^{\text{th}}$ covariance component:

$$B_0(0) = \sum_{l=-L}^{L} R_{1l}^{(\xi)}(0) = \frac{1}{4}\sum_{l=-L}^{L}\left[R_{\xi_l}^c(0) + R_{\xi_1}^s(0)\right],$$

For the covariance components of positive order $k > 0$,we have:

$$B_k(0) = \frac{1}{4}\sum_{l=k-L}^{L}\left[R_{\xi_{l-k}\xi_L}^c(0) + R_{\xi_{l-k}\xi_l}^s(0) - i\left[R_{\xi_{l-k}\xi_l}^{cs}(0) - R_{\xi_{l-k}\xi_l}^{sc}(0)\right]\right]. \qquad (13)$$

As we can see from these relations, the first covariance component is determined by the sum of the elements of the first upper diagonal, the second covariance component – by the sum of the elements of the second upper diagonal, etc. Similarly, the covariance components with negative $k < 0$ are determined by the respective sums of the elements for the diagonals of the matrix bottom part.

The matrix, whose elements are modulus

$$\left|R_{kl}^{(\xi)}(0)\right| = \frac{1}{4}\left[\left[R_{\xi_k \xi_l}^c(0) + R_{\xi_k \xi_l}^s(0)\right]^2 + \left[R_{\xi_k \xi_l}^{cs}(0) + R_{\xi_k \xi_l}^{sc}(0)\right]^2\right]^{1/2},$$

is symmetric, therefore the sums of the elements for the respective upper and bottom diagonals are the same:

$$\sum_{l=k-L}^{L} \left| R_{l-k,l}^{(\xi)}(0) \right| = \sum_{l=-L}^{k+L} \left| R_{l-k,l}^{(\xi)}(0) \right| .$$

Then, for characterizing the cross-covariances of the modulating processes, we can choose the quantity

$$S_k = 2 \sum_{l=k-L}^{L} \left| R_{l-k,l}^{(\xi)}(0) \right| .$$

It follows from Eq. (13) that $2|B_k(0)| < S_k$.

Proceeding from the values of the elements for matrix $\left[\left| R_{kl}^{(\xi)}(0) \right| \right]_{2L \times 2L}$, we can evaluate the specific features of the auto- and cross-covariances for modulating processes $\xi_k(t)$ and the contribution of each pair to the covariance component values.

A similar decomposition will also be useful for the analysis of the dependences of the covariance components on lag. Based on Eq. (11), we can interpret and parameterize the empirical dependences, which are obtained by the experimental data processing.

CONCLUSIONS

The properties of the mean and covariance function in the PNRP are determined by the properties of their modulating processes. The mathematical expectations of PNRS are equal to the Fourier coefficients of the mean function. The cross-covariance functions determine the Fourier coefficients of the PNRP covariance function with different numbers. The random process is the second order periodically non-stationary only in the case when some of the cross-covariance functions of its modulation processes are not equal to zero. The correlations of the PNRP spectral harmonics and the correlations of the modulating processes in series representation are equivalent.

Evaluating the specific features of the auto- and cross-covariances for modulating processes and the contribution of each pair to the covariance component values allows us to detect defects at early stages.

1. Dragan Y.; Javorskyj I. *Rhythmics of sea waving and underwater acoustic signals*; Naukova Dumka, 1982 (in Russian).
2. Gardner, W.A. *Cyclostationarity in Communications and Signal Processing;* IEEE Press, 1994.
3. Antoni, J. Cyclostationarity by examples, *Mech. Syst. Signal Process.*, **2009**, 23, 987–1036. https://doi.org/10.1016/j.ymssp.2008.10.010
4. Javorskyj, I.; Yuzefovych, R.; Matsko, I.; Kurapov, P. Hilbert transform of a periodically non-stationary random signal: Low-frequency modulation, *Digit. Signal Process*., **2021**, 116, 103113. https://doi.org/10.1016/j.dsp.2021.103113
5. McFadden, P.D.; Smith, J.D. Vibration monitoring of rolling element bearings by the high frequency resonance technique – A review, *Tribol. Int.,* **1984**, 17, 3–10. https://doi.org/10.1016/0301-679X(84)90076-8
6. Ho, D.; Randall, R.B. Optimization of bearing diagnostic techniques using simulated and actual bearing fault signals, *Mech. Syst. Signal Process*., **2000**, 14, 763–788. https://doi.org/10.1006/mssp.2000.1304
7. Antoni, J. Cyclic spectral analysis of rolling-element bearing signals: Facts and Fictions, *J. Sound Vib*., **2007**, 304, 497–529. https://doi.org/10.1016/j.jsv.2007.02.029
8. Randall, R.B.; Antoni, J. Rolling element bearing diagnostics – A tutorial, *Mech. Syst. Signal Process*., **2011**, 25, 485–520. https://doi.org/10.1016/j.ymssp.2010.07.017
9. Napolitano, A. *Cyclostationary processes and time series: Theory, applications, and generalizations*; Elsevier, 2020.
10. Mykhailyshyn, V.; Javorskyj, I.; Vasylyna, Y.; Drabych, O.; Isaev, I. Probabilistic models and statistical methods for the analysis of vibration signals in the problems of diagnostics of machines and structures, *Materials Science,* **1997**, 33, 655–672. https://doi.org/10.1007/BF02537594
11. McCormick, A.C.; Nandi, A.K. Cyclostationarity in rotating machine vibrations, *Mech. Syst. Signal Process.,* **1998**, 12, 225–242. https://doi.org/10.1006/mssp.1997.0148

12. Capdessus, C.; Sidahmed, M.; Lacoume, J.L. Cyclostationary processes: Application in gear fault early diagnostics, *Mech. Syst. Signal Process*., **2000**, 14, 371–385. https://doi.org/10.1006/mssp.1999.1260
13. Antoniadis, I.; Glossiotis, G. Cyclostationary analysis of rolling-element bearing vibration signals, *J. Sound Vib*., **2001**, 248, 829–845. https://doi.org/10.1006/jsvi.2001.3815
14. Antoni, J.; Bonnardot, F.; Raad, A.; El Badaoui, M. Cyclostationary modeling of rotating machine vibration signals, *Mech. Syst. Signal Process*., **2004**, 18, 1285–1314. https://doi.org/10.1016/S0888-3270(03)00088-8
15. Antoni, J.; Randall, R.B. A stochastic model for simulation and diagnostics of rolling element bearings with localized faults, *ASME J. Vib. Acoust.*, **2003**, 125, 282–289 https://doi.org/10.1115/1.1569940
16. Javorskyj, I.; Kravets, I.; Matsko, I.; Yuzefovych, R. Periodically correlated random processes: Application in early diagnostics of mechanical systems, *Mech. Syst. Signal Process*., **2017**, 83, 406–438. https://doi.org/10.1016/j.ymssp.2016.06.022
17. Borghesani, P.; Pennacchi, P.; Ricci, R.; Chatterton, S. Testing second order cyclostationarity in the squared envelope spectrum of non-white vibration signals, *Mech. Syst. Signal Process*., **2013**, 40, 38–55. https://doi.org/10.1016/j.ymssp.2013.05.012
18. Wang, H. Early detection of gear tooth cracking using the resonance demodulation technique, *Mech. Syst. Signal Process*., **2001**, 15, 887–903. https://doi.org/10.1006/mssp.2001.1416
19. Kay, S.M. *Modern spectral estimation: Theory and application*. Prentice Hall, 1988.
20. Randall, R.B.; Sawalhi, N.: Coats, M. A comparison of methods for separation of deterministic and random signals, *Int. J. Cond. Monitoring,* **2011**, 1, 11–19. https://doi.org/10.1784/204764211798089048
21. Borghesani, P.; Pennacchi, P.; Randall, R.B.; Sawalhi, N.; Ricci, R. Application of cepstrum pre-whitening for the diagnosis of bearing faults under variable speed conditions, *Mech. Syst. Signal Process.*, **2013**, 36, 370–384. https://doi.org/10.1016/j.ymssp.2012.11.001
22. Hurd, H.L.; Miamee, A. *Periodically Correlated Random Sequences: Spectral Theory and Practice*; Wiley, 2007. https://doi.org/10.1002/9780470182833
23. Antoni, J.; Borghesani, P. A statistical methodology for the design of condition indicators, *Mech. Syst. Signal Process.*, **2019**, 114, 290–327. https://doi.org/10.1016/j.ymssp.2018.05.012
24. Wiener, N. Generalized harmonic analysis, *Acta Math.*, **1930**, 55, 117–258. https://doi.org/10.1007/BF02546511
25. Javorskyj, I.; Isayev, I.; Majewski, J.; Yuzefovych, R. Component covariance analysis for periodically correlated random processes, *Signal Process.*, **2010**, 90(4), 1083–1102. https://doi.org/10.1016/j.sigpro.2009.07.031

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